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EMERGENCE OF ORDERED MOTION OF THE OSCILLATOR DRIVEN BY FLUCTUATING FORCE

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Abstract: Computational experiments with double pendulum, Tacker's oscillator and steel beam, described by Duffing equations, are performed. We assume that a fluid drives the oscillator by fluctuating force. The considered complex motion is a combination of deterministic chaos and stochasticity. If amount of the fluctuating force is large enough (the number of fluid particles interacting with the oscillator is then large), oscillator motion becomes ordered. Similar result is obtained in the Lorenz model, when considering a part of the Earth atmosphere interacting with surrounding air.
Keywords: double pendulum, chaos, stochasticity, emergence.

1. INTRODUCTION

Emergence is a collective behavior of a large number (which approaches infinity) of microscopic parts, qualitatively different from behavior of individual parts. Laminar and turbulent fluids flow are examples of emergent phenomena [1]. Vince Darley defines emergent phenomena as one for which simulation is the optimal way of prediction [2]. Emergence of order is considered as topic of biology, geology, physics, chemistry, engineering and mathematics [3].

Ferromagnetism, non-conventional superconductivity and mechanical properties of graphene, quasiparticles and arrow of time are emergents [4]. Interaction of X-rays, electrons, neutrons and probes with superlattices $PbTiO_3 / SrTiO_3$ causes emergence of new phases [5]. Life [6] and macroeconomy [7] are emergents.

If an emergent phenomenon is in principle reducible to microphysics, even though the behavior of the whole cannot be determined by the behavior of the parts, then this is a weak emergence. If emergent properties generally cannot be reduced to microphysical properties, then this is a strong emergence [6].

The most prominent researchers of emergent phenomena are Philip Anderson [8] and Robert Laughlin [9,10]. They considered moving physics away from reductionism necessary. Complexity is understood if we understand emergence - the appearance of properties on a larger scale unrelated to the properties of parts of the system. This is a picture of the world significantly different from the reductionist picture of Paul Dirac's world, which implies a mathematical description of all phenomena based on fundamental laws. Philip Anderson, unlike Paul Dirac, considers the laws of solid state physics as fundamental as laws of particle physics.

Here we observe complex motion of a double pendulum [11], Thacker oscillator [12], steel bar in the Duffing model [13] and the Earth's atmosphere in the Lorenz model [14]. In all four cases, we assume the action of a fluctuating force (Figure 1). Complex motion is a combination of deterministic chaos and stochasticity. While chaos is predictable in the short term, stochasticity is completely unpredictable. We will show that as the number of interacting particles increases, accompanied by an increase in the intensity of the fluctuating force, the order of motion increases.

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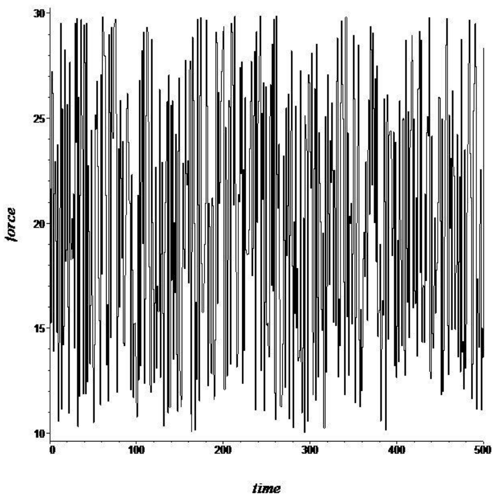

Figure 1. Fluctuating force $F_f(t)$, with certain realization of series of random numbers in a certain interval. For same interval, various realizations of a series of random numbers are possible. Time step is $h = 0.05$.

2. DOUBLE PENDULUM

Two bars, AB and BC, whose masses are m_1 and m_2, lengths L_1 and L_2, are oscillating around the axes passing through points A and B. We solve differential equations

$$\frac{d\alpha}{dt} = \omega_a(t) \tag{2.1}$$

$$\frac{d\beta}{dt} = \omega_b(t) \tag{2.2}$$

$$\begin{aligned}&(m_1+m_2)L_1\frac{d\omega_a}{dt} + m_2L_2\frac{d\omega_b}{dt}\cos(\alpha-\beta) + m_2L_2\omega_b^2\sin(\alpha-\beta) + \\ &+ g(m_1+m_2)\sin\alpha + \kappa(\omega_a+\omega_b) - F_f = 0\end{aligned} \tag{2.3}$$

$$m_2L_2\frac{d\omega_b}{dt} + m_2L_1\frac{d\omega_a}{dt}\cos(\alpha-\beta) - m_2L_1\omega_a^2\sin(\alpha-\beta) + gm_2\sin\beta + \kappa(\omega_a+\omega_b) - F_f = 0 \tag{2.4}$$

where α and β are angular displacements, ω_a and ω_b angular velocities of rods with respect to first and second axes and κ is a damping ratio [11]. We calculate with

$$m_1 = 4.23, m_2 = 2.13, L_1 = 1.14, L_2 = 2.24, \alpha(0) = 1.49, \beta(0) = 2.99, \omega_a(0) = \omega_b(0) = 0, \kappa = 0.6\,. \tag{2.5}$$

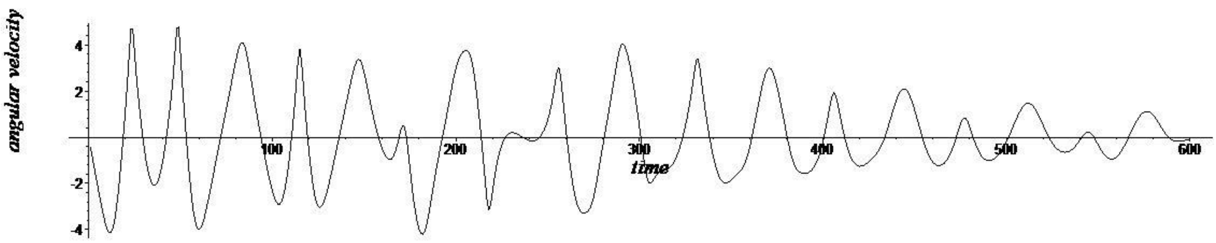

Figure 2.1 Angular velocity of the first pendulum for $-3 \le F_f \le -2.7$. Unit of time is $h = 0.05$.

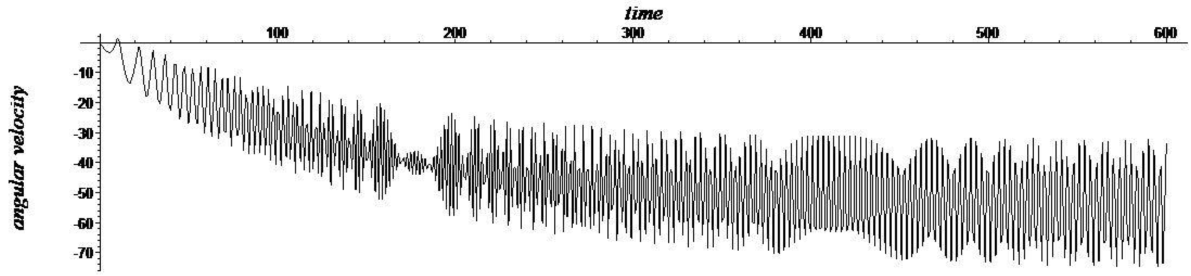

Figure 2. Angular velocity of the first pendulum for $-70 \le F_f \le -63$.

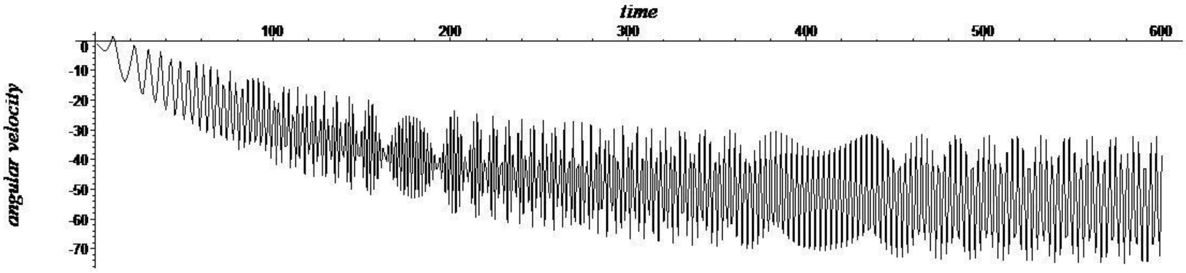

Figure 3. Angular velocity of the first pendulum for $-70 \le F_f \le -63$ *(second realization of a series of random numbers). After a long enough period approximate periodicity appears.*

3. THACKER OSCILLATOR

Thacker oscillator is used in laboratory exercises in non-linear dynamics course [12]. Magnet with dipole moment μ is inserted in a magnetic field B_f which is generated by one pair of coils and magnetic field $B_d \sin\omega_d t$ which is generated by second pair of coils (directions of these two fields are normal to each other). We solve equations

$$\frac{d\varphi}{dt} = \omega(t) \tag{3.1}$$

$$\frac{d\omega}{dt} = \mu(B_f \sin\varphi + B_d \cos\varphi \sin\omega_d t) - \kappa\omega + F_f \tag{3.2}$$

where φ is angular displacement of a magnet and κ is damping ratio. We assume

$$\mu = 7.3, B_f = 1.8, B_d = 8.1, \omega_d = 1.8, \kappa = 0.8, \varphi(0) = -5.1, \omega(0) = 7.1 \tag{3.3}$$

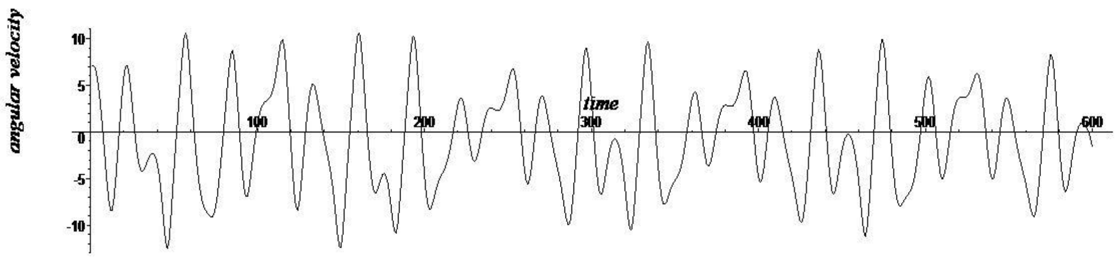

Figure 4. Angular velocity of the Thacker oscillator for $-7 \le F_f \le -6.3$.

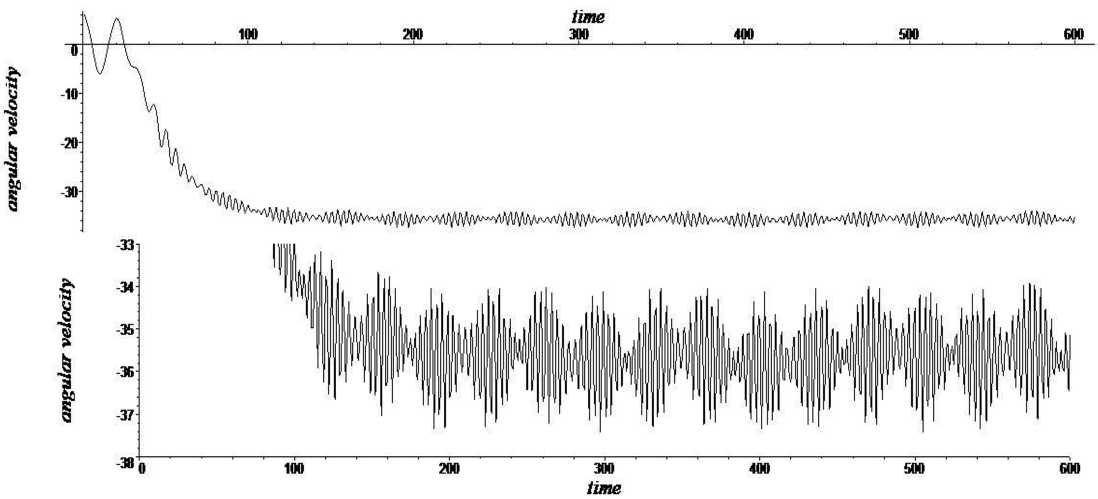

Figure 5. Angular velocity of the Thacker oscillator for $-30 \le F_f \le -27$.

4. DUFFING OSCILLATOR

Oscillations of a steel bar are described with equations [13]

$$\frac{dx}{dt} = v(t) \quad (4.1)$$

$$\frac{dv}{dt} = 6.9x - 11.74x^3 - 0.02v + 3.2\sin 9.1t + F_f \quad (4.2)$$

which we will solve with initial conditions $x(0) = -1.9, v(0) = 0.34$.

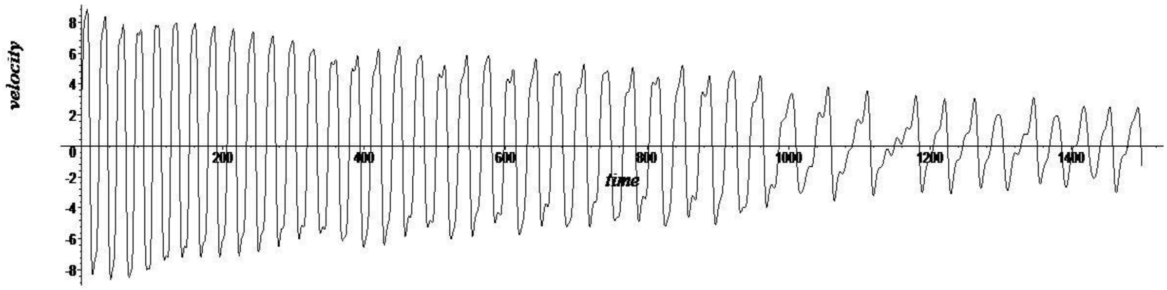

Figure 6. Velocity of the Duffing oscillator for $2.7 \le F_f \le 3.0$.

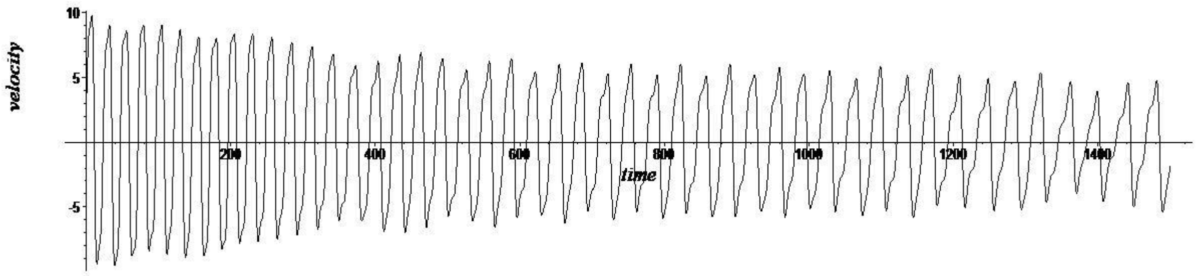

Figure 7. Velocity of the Duffing oscillator for $5.4 \le F_f \le 6.0$.

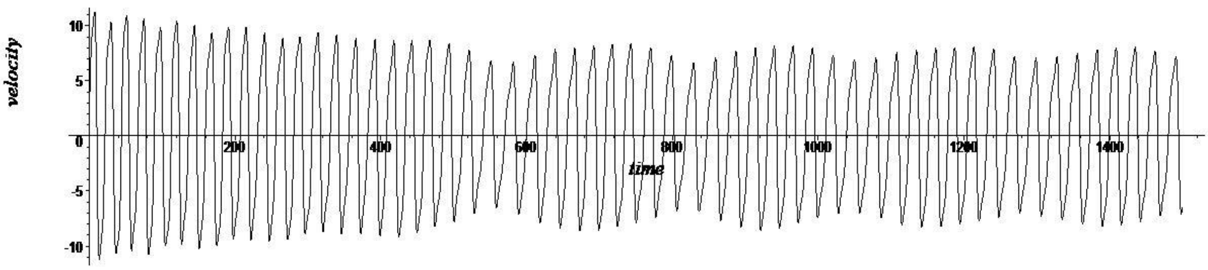

Figure 8. Velocity of the Duffing oscillator for $10.8 \le F_f \le 12.0$.

5. LORENZ MODEL

We assume that part of the Earth's atmosphere is acted by fluctuating force $F_f(t)$ and we solve equations

$$\frac{dx}{dt} = \sigma(y - x) + F_f$$

$$\frac{dy}{dt} = rx - y - xz \quad (5.1)$$

$$\frac{dz}{dt} = xy - bz$$

where x is convection velocity (this is why F_f is in first equation of these systems of equations), y is horizontal component of temperature gradient, z is vertical component of temperature gradient, σ is Prandtl number, r is Rayleigh number, b is ratio of dimensions of layers of fluid [14]. In original Lorenz model $F_f = 0$ is assumed. We assume

$$\sigma = 10, r = 28.3, b = 8/3, x(0) = -19.1, y(0) = 21.5, z(0) = -17.8.$$

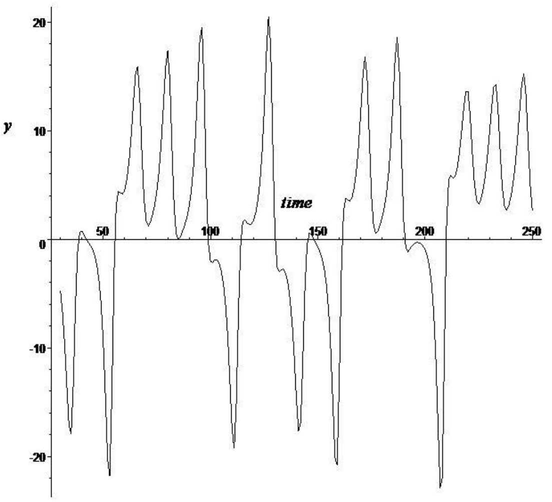

Figure 9. $y(t)$ *for* $-10 \le F_f \le 0$.

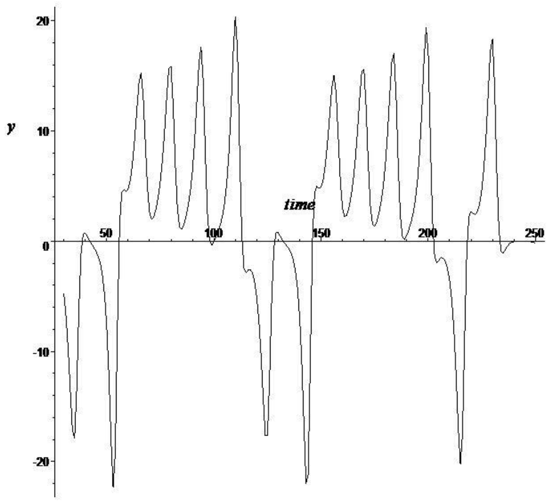

Figure 10. $y(t)$ *for* $0 \le F_f \le 10$.

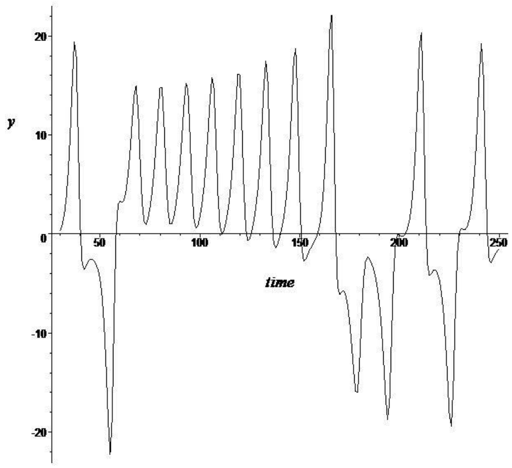

Figure 11. $y(t)$ *for* $10 \le F_f \le 20$.

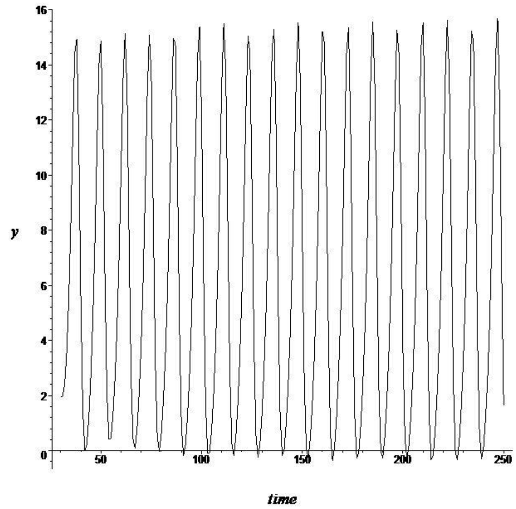

Figure 12. $y(t)$ *for* $20 \le F_f \le 30$ *(approximate periodicity).*

6. CONCLUSION

We considered four seemingly very different systems - a double pendulum, a rod magnet that rotates under the influence of a changing magnetic field, a steel rod that oscillates and part of the Earth's atmosphere. The mathematical descriptions of these systems, (2.1)-(2.4), (3.1)-(3.2), (4.1)-(4.2) and (5.1) are very different. However, our computer experiments show that the emergence of order occurs in all these systems in the same way, by increasing the intensity of the fluctuating force acting on them. Approximate periodic motion occurs after a sufficiently long time if the number of particles the system is made of and the number of fluid particles acting on the system are large enough. There seems to be a universal way of combining deterministic chaos and stochastics that allows the emergence of order.

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ЕМЕРГЕНЦИЈА УРЕЂЕНОГ КРЕТАЊА ПРИ ДЈЕЛОВАЊУ ФЛУКТУИРАЈУЋЕ СИЛЕ НА ОСЦИЛАТОР

Сажетак: Направљени су рачунарски експерименти са двоструким клатном, Такеровим осцилатором и челичном шипком, чије осциловање је описано Дафинговим једначинама. Претпоставља се да флуид дјелује флуктуирајућом силом на осцилатор. Посматрамо сложено кретање, које је комбинација детерминистичког хаоса и стохастике. Кад је интензитет флуктуирајуће силе довољно велик (тад је број честица флуида са којима осцилатор међудјелује велик), кретање осцилатора постаје уређено. Сличан резултат добије се у Лоренцовом моделу, кад дио Земљине атмосфере међудјелује са околним ваздухом.

Кључне ријечи: двоструко клатно, хаос, стохастика, емергенција.

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